

# New Spaces in Mathematics and Physics

Formal and conceptual reflections

Collective Book Project

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## Editors

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## Description of the project

Since the development of differential and algebraic varieties, the notion of space has traversed very radical transformations, both in pure mathematics and in mathematical physics. The objective of this project is to propose a global vision of these recent evolutions by editing a collective book primarily addressed to mathematicians and physicists, but also to historians and philosophers of these disciplines.

Each chapter would address a particular notion  $X$  of space in mathematics and/or physics (e.g.  $X$  = scheme, topos, stack, spin network, homotopy types, noncommutative space, supermanifolds, etc.). A preliminary version of the contents for the book is given below. Each contributor is invited to write a chapter proposing a *formal introduction* as well as a *conceptual discussion* of the corresponding notion. We do not intend each chapter to be a comprehensive treatment of the subject in question but rather an opportunity to introduce it to a wider audience and to present aspects of the subject that are usually absent in textbooks. For instance, the contributors are invited to address some of the following questions: What is the formal, conceptual and/or historical path that led to the emergence of the notion  $X$ ? What are the basic intuitions or ideas underpinning the corresponding formalization? What are the technical and conceptual advantages of the notion  $X$  with respect to other notions? Does the notion  $X$  establish new relations between geometry and other branches of mathematics and/or physics? What are the new perspectives opened by the notion  $X$  and the new frontiers that it delineates?

## Conference at IHP

As a part of this project, we have organized a conference on the matter of the book at the Institut Henri Poincaré in Paris from the 28th of september to the 2nd of october 2015. The conference was conceived as an opportunity for the different authors to meet and to enrich the different contributions to the book.

All the information about this event, including videos of all the talks can be found online here: <https://ercpqg-espace.sciencesconf.org>.

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